O(g) corrections to the transport coefficients of QCD

Jacopo Ghiglieri, CERN

In collaboration with Guy Moore and Derek Teaney

QCD in Finite Temperature and Heavy-Ion Collisions
BNL, February 14th 2017

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Overview

- Aim: compute the transport coefficients of QCD to NLO. LO is AMY Arnold Moore Yaffe 2003
- NLO means $O(g)$ effects from the medium
- Relies on cool new light-cone techniques (much more complicated for non-relativistic or mildly relativistic degrees of freedom)
  Pedagogical review of the techniques in
  JG Teaney **1502.03730**
  Most of the ingredients (kinetic theory to NLO) in
  JG Moore Teaney **1509.07773** (my talk at QM15)

Motivation

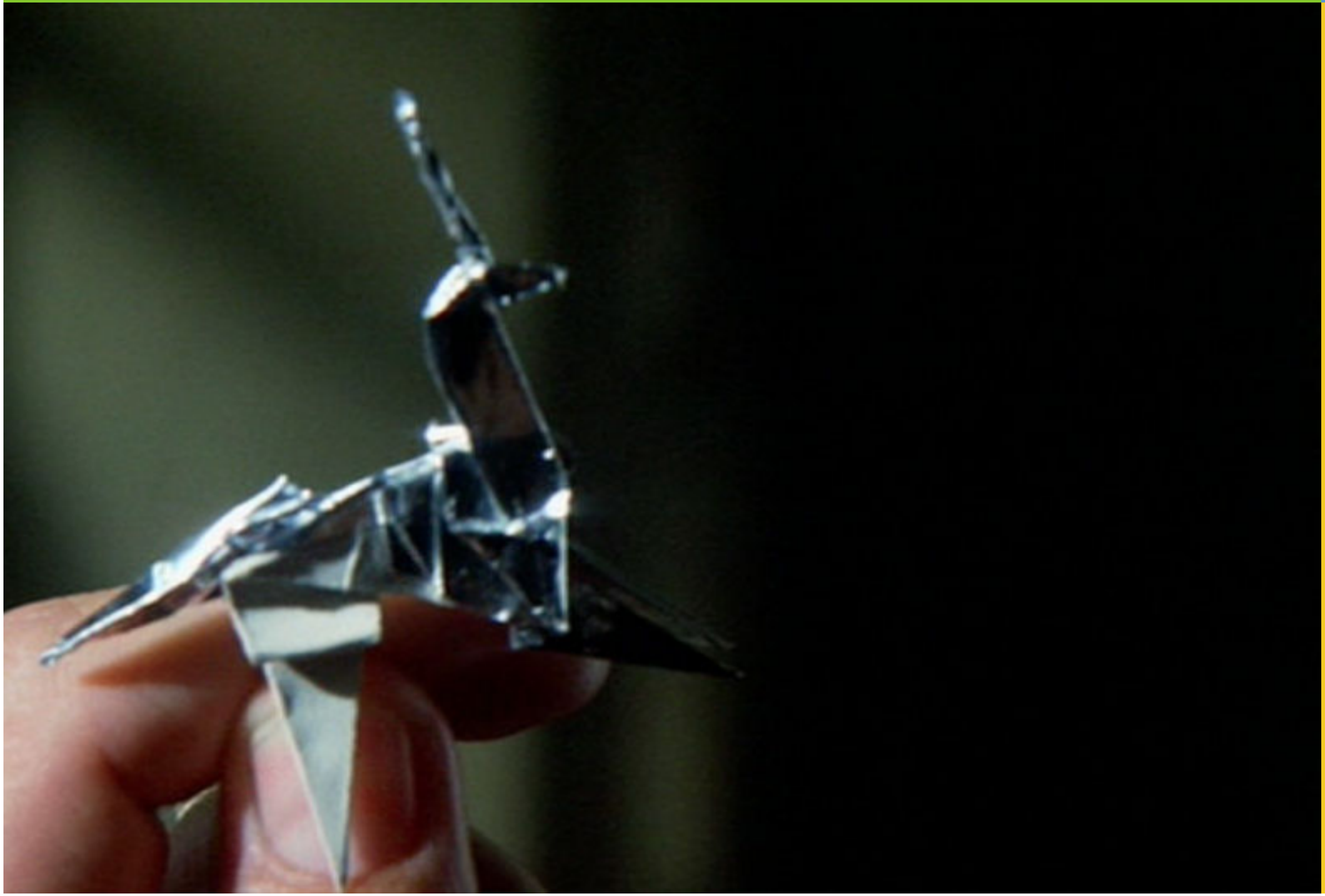
- How reliable is pQCD when extrapolating to α_s=0.3?
  - For thermodynamical quantities (p, s, ...) either strict expansion in g (QCD (T) + EQCD (gT) + MQCD (g^2T) Arnold-Zhai, Braaten Nieto, etc), or non-perturbative solution of EQCD (Kajantie Laine etc.) or resummations (HTLpt, Andersen Braaten Strickland etc.)
  - For dynamical quantities? We now have 2 contrasting examples of $O(g)$ corrections: very large for momentum diffusion (heavy quarks Caron-Huot Moore (2007), $\hat{q}$ Caron-Huot (2008)), reasonable (~20%) for e.m. probes (JG *et al.*, Laine, Laine Ghisoiu (2013-14))

Outline

- ✓ Introduction and motivation
- Theory overview, slightly less time-constrained*
- Results for the shear viscosity and quark number diffusion
- Conclusions

* More details in the backup slides and in the upcoming papers
This symbol:  interesting but having to skip for lack of time

Theory overview



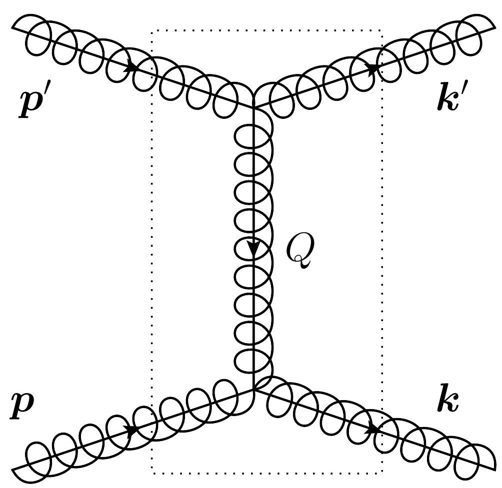
The AMY kinetic theory

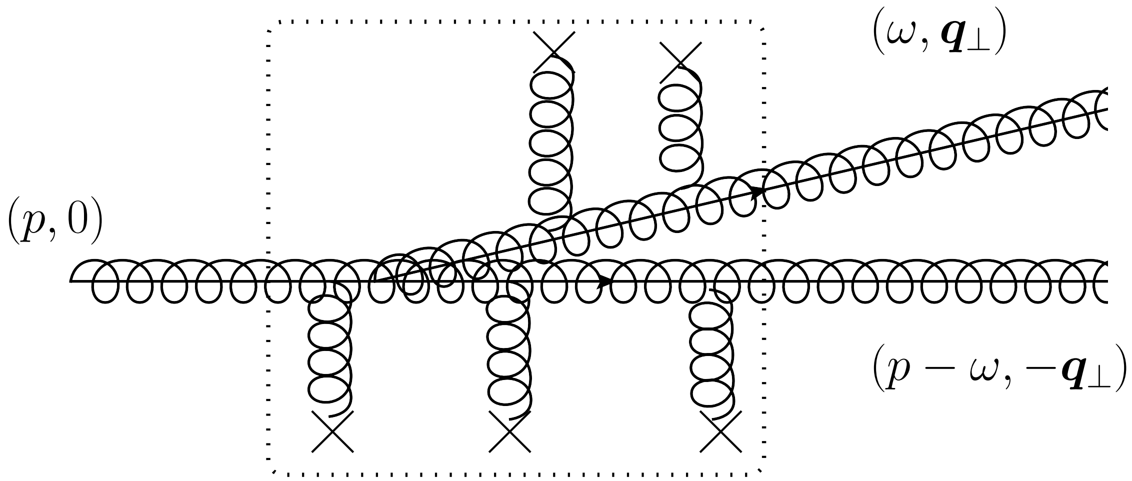
The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, AMY, all that) AMY (2003)





Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\rm EQ}(\mathbf{p}) + \sum_\ell \delta f_\ell(\mathbf{p}) \qquad \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\rm EQ}(\mathbf{p}, u, \beta, \mu) = C_{\rm lin}[\delta f_\ell]$$

- Driving term equates linearized collision operator. Since $\langle T^{i\neq j}\rangle \propto \eta$, $\langle \mathbf{J}_q\rangle = -D_q \boldsymbol{\nabla}\langle n_q\rangle$ η requires ℓ=2, D_q ℓ=1

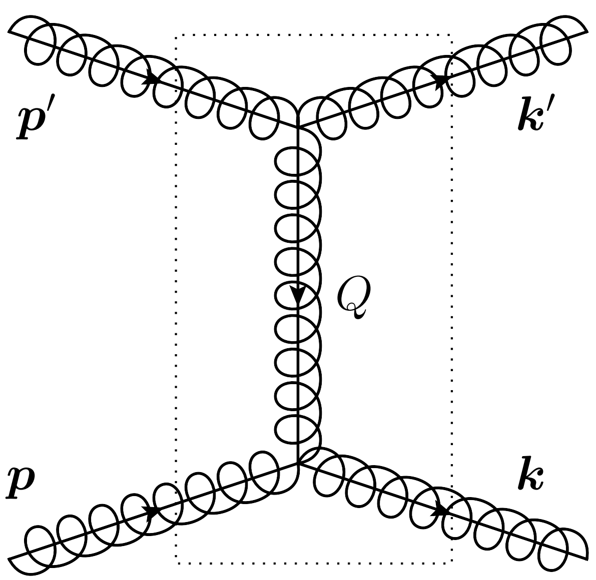
- Transport coefficients obtained by the kinetic thy definitions of T, J once δf_ℓ has been obtained. Solution easier in **quadratic form** (variational). LO η,D~$1/g^4$

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\rm EQ}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p})\, C_{\rm lin}[\delta f_\ell]$$

Arnold Moore Yaffe (2003)

Reorganization

- The NLO corrections come from regions sensitive to soft gluons (no quarks in this illustration)
- Before we get there, let's have a reorganized perspective on these regions at LO
- Look at 2↔2 scattering

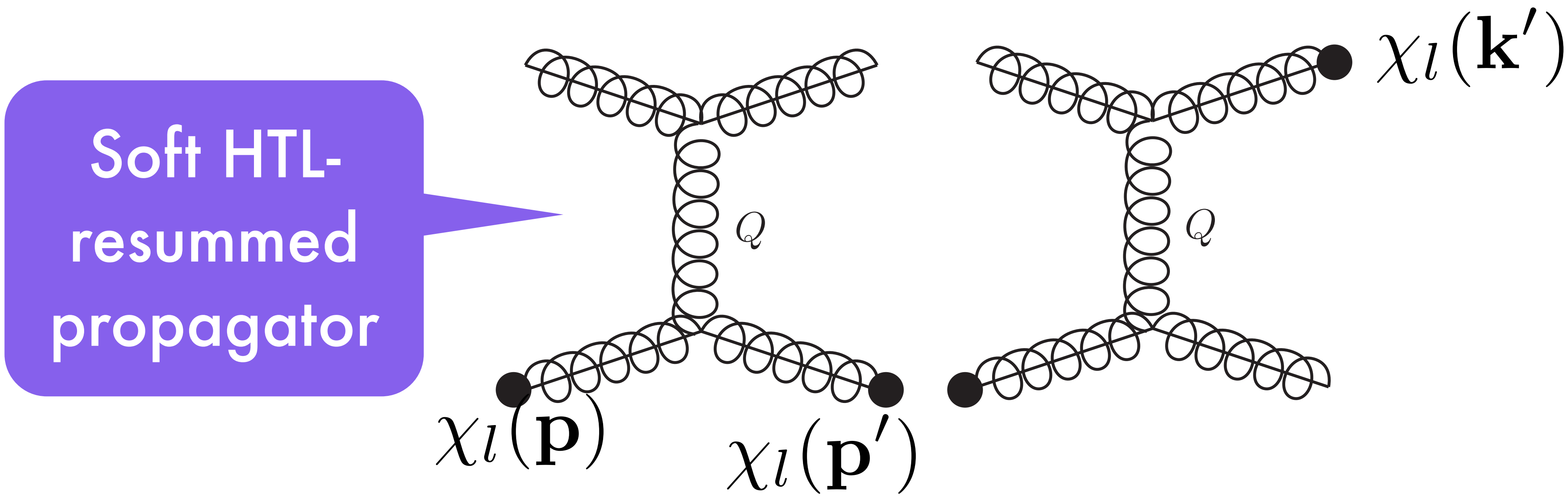


$$\int_{\mathbf{pkp'k'}} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}',\mathbf{k}')\right|^2 (2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\, f_{\rm EQ}(p)\, f_{\rm EQ}(k)\,[1+f_{\rm EQ}(p')]\,[1+f_{\rm EQ}(k')]$$
$$\times \Big[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\Big]^2$$

$$\delta f_l(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,\chi_l(\mathbf{p})$$

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- Left: diffusion terms, **p** and **p′** strongly correlated

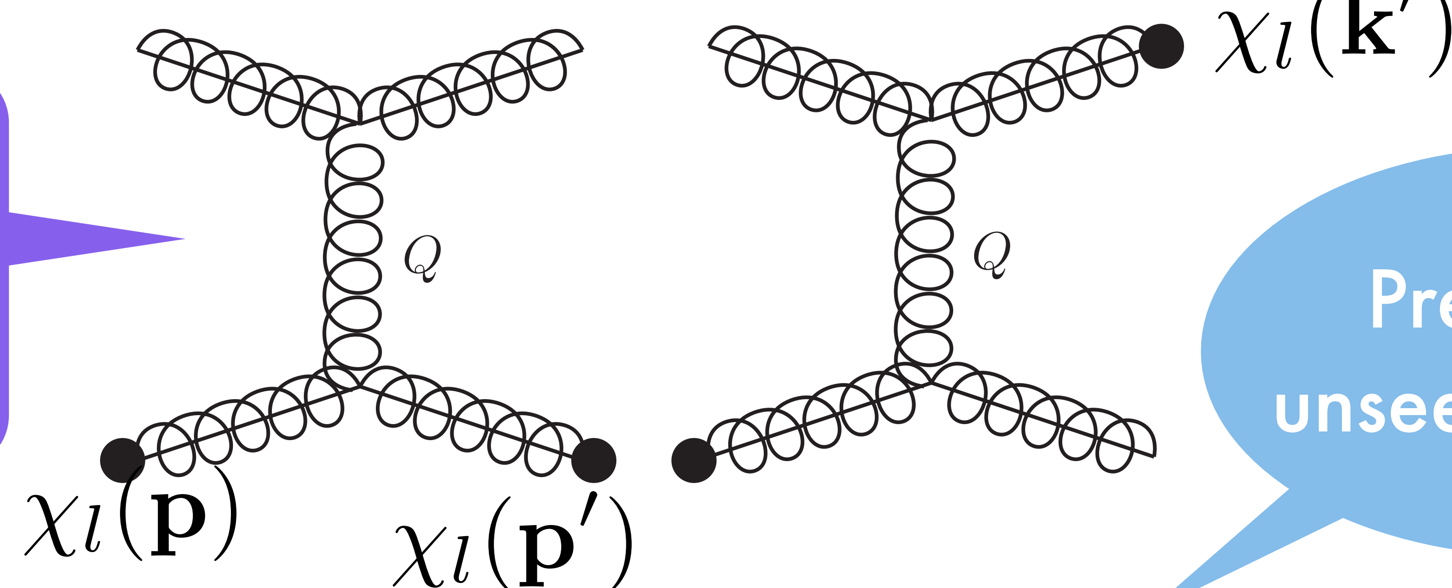
$$\left(\chi_\ell(\mathbf{p})-\chi_\ell(\mathbf{p}')\right)^2=(\hat{\mathbf{p}}\cdot\mathbf{q})^2[\chi'(p)]^2+\frac{\ell(\ell+1)}{2}\frac{q^2-(\hat{\mathbf{p}}\cdot\mathbf{q})^2}{p^2}[\chi(p)]^2$$

identify a longitudinal and a transverse momentum broadening contribution, $\hat{q}_L$ and $\hat{q}$

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$

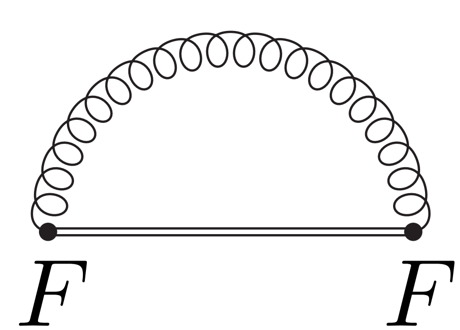
Soft HTL-resummed propagator



Previously unseen footage

- **Diffusion terms:** transverse becomes Euclidean

$$\hat{q}(\mu_\perp)=g^2C_A\int^{\mu_\perp}\frac{d^2q_\perp}{(2\pi)^2}\int\frac{dq^+}{2\pi}\langle F^{-\perp}(Q)F^{-}{}_{\perp}\rangle_{q^-=0}$$
$$=g^2C_AT\int^{\mu_\perp}\frac{d^2q_\perp}{(2\pi)^2}\frac{m_D^2}{q_\perp^2+m_D^2}=\frac{g^2C_ATm_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

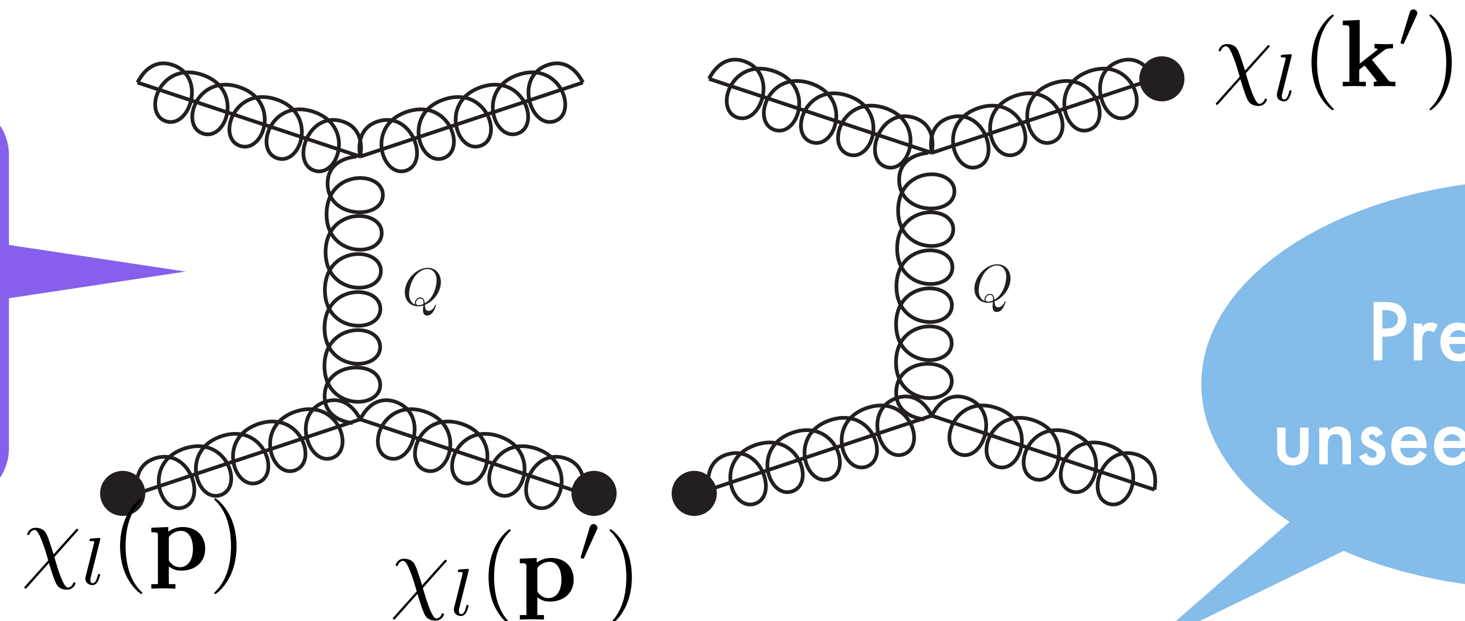


Aurenche Gelis Zaraket **JHEP0205** (2002), Caron-Huot **PRD79** (2009)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$

Soft HTL-resummed propagator

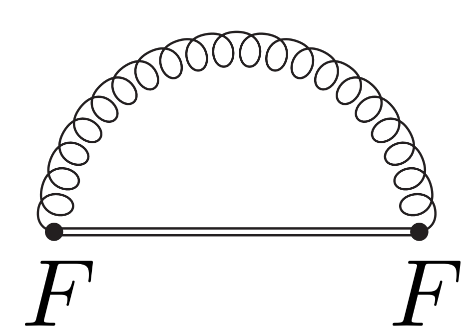


Previously unseen footage

- **Diffusion terms:** longitudinal with lightcone sum rule

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-z}(Q) F^{-z} \rangle_{q^-=0}$$

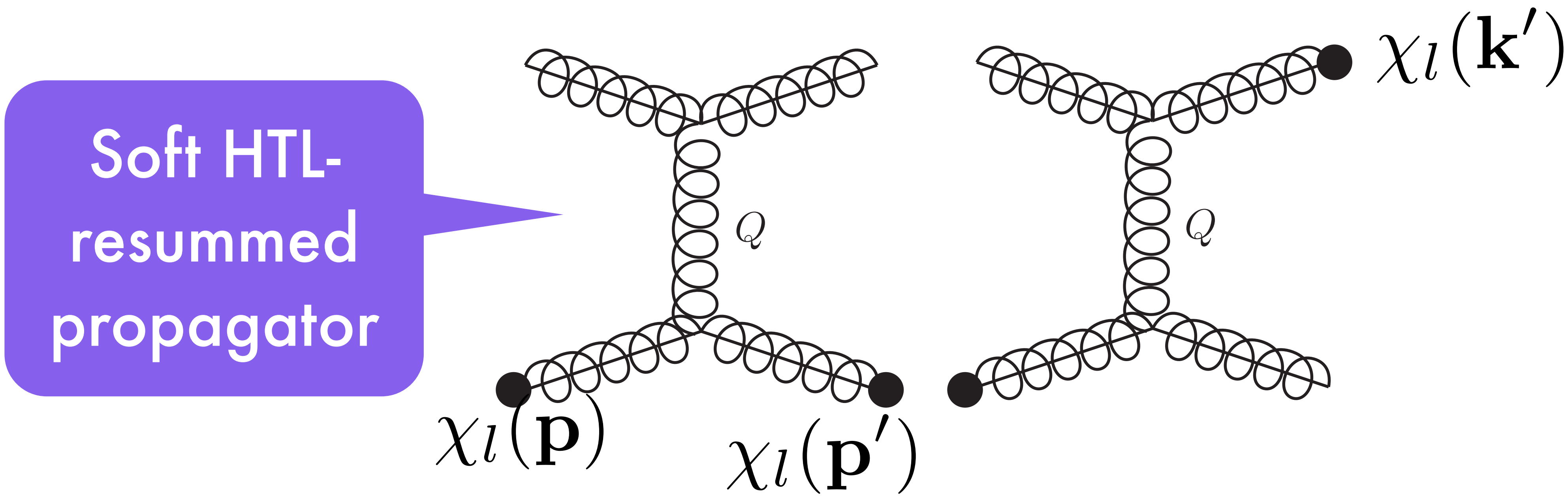
$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2} = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln \frac{\mu_\perp}{m_\infty}$$



JG Moore Teaney (2015)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- **Diffusion terms:** easy with light-cone techniques*

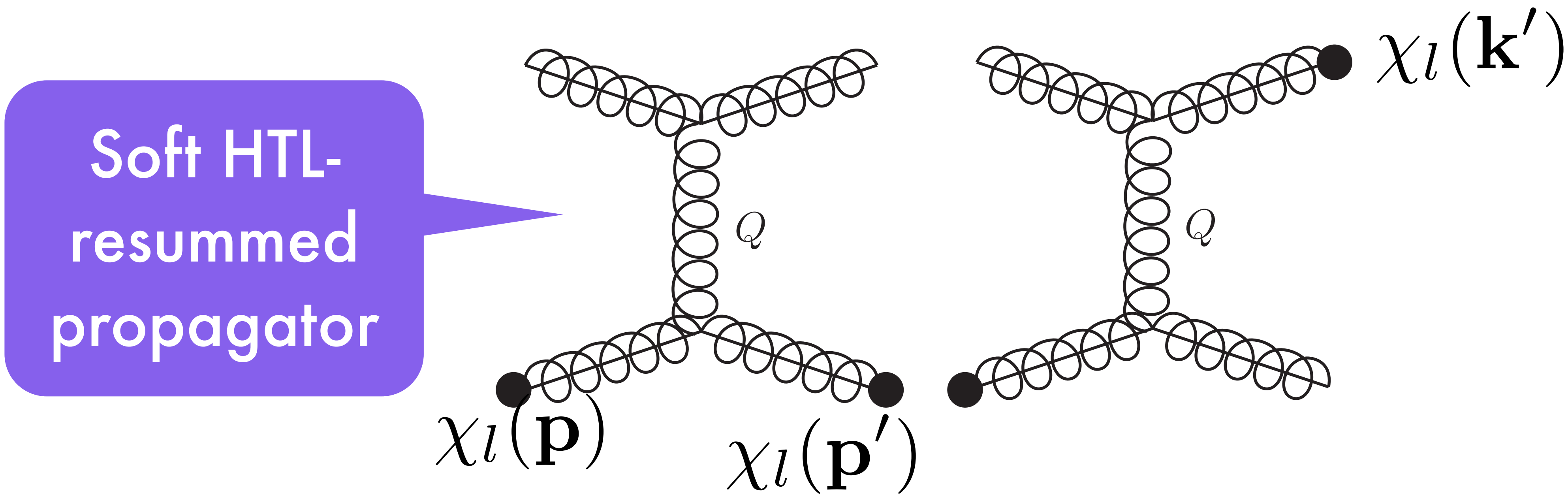
$$\hat{q}_L^a\Big|_{\mathrm{soft}}=\frac{g^2C_{R_a}Tm_D^2}{4\pi}\ln\frac{\sqrt{2}\mu_\perp}{m_D}\qquad \hat{q}^a\Big|_{\mathrm{soft}}=\frac{g^2C_{R_a}Tm_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

give rise to the leading log contribution

*Caron-Huot PRD82 (2008) JG Moore Teaney (2015)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$

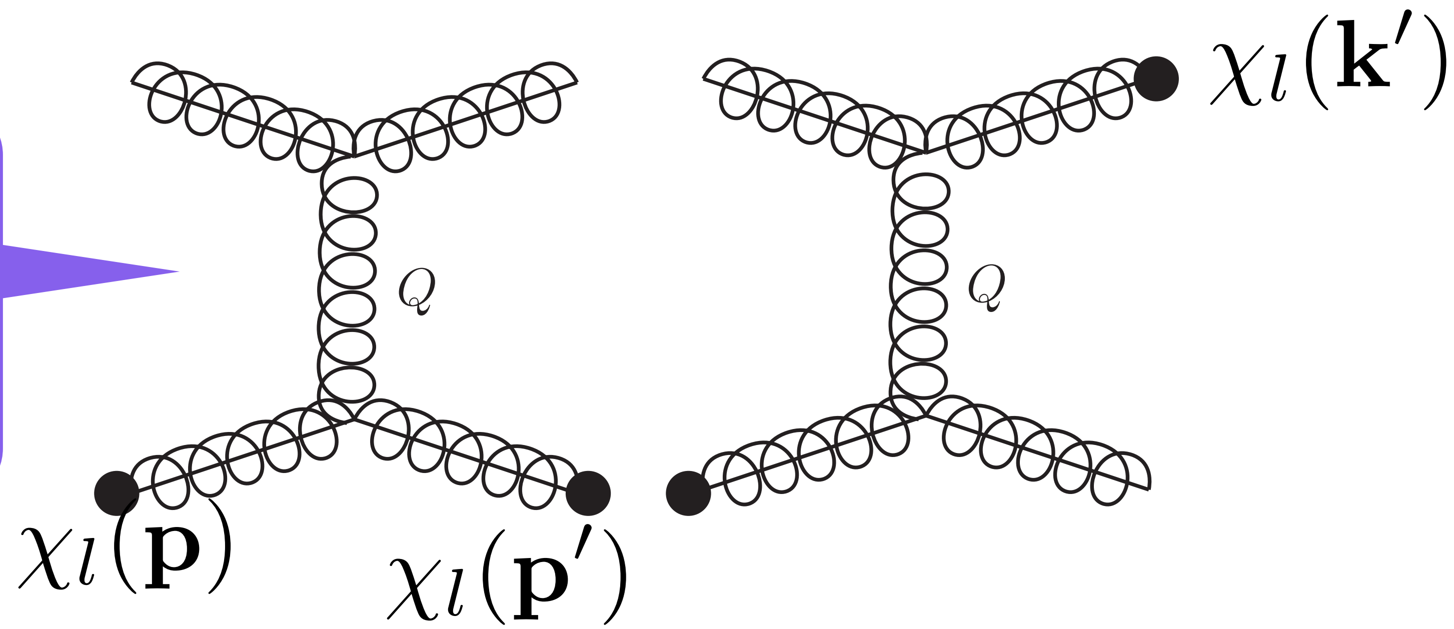


- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Two-point function of two uncorrelated deviations from equilibrium
(diffusion was the response of an off-eq leg to the equilibrium bath)

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$

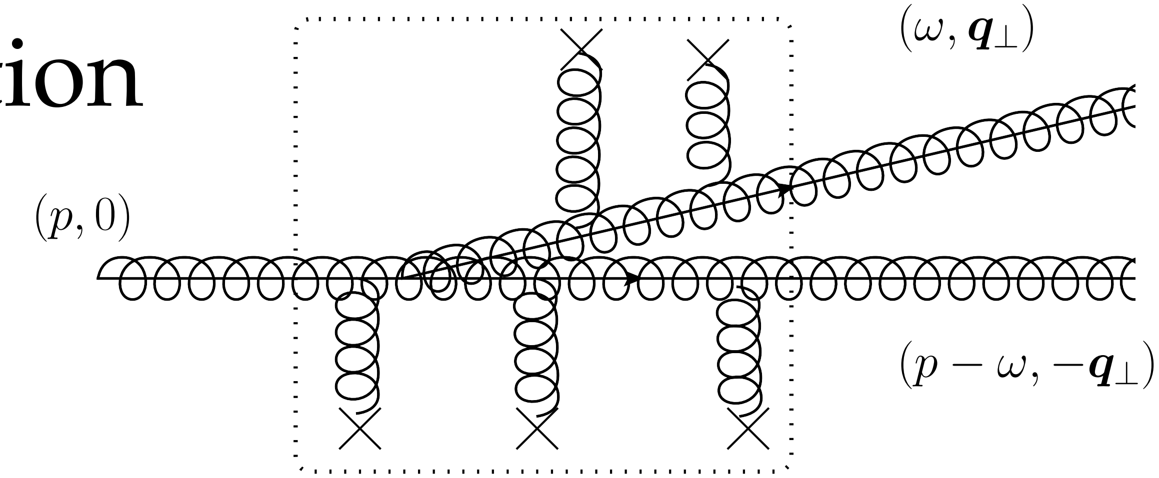
Soft HTL-resummed propagator



- Right: cross terms, **p**,**p'** and **k**,**k'** not correlated. Light-cone techniques not applicable, have to use numerical integration. Easy at LO, where they are finite (no leading log contribution)

Reorganization

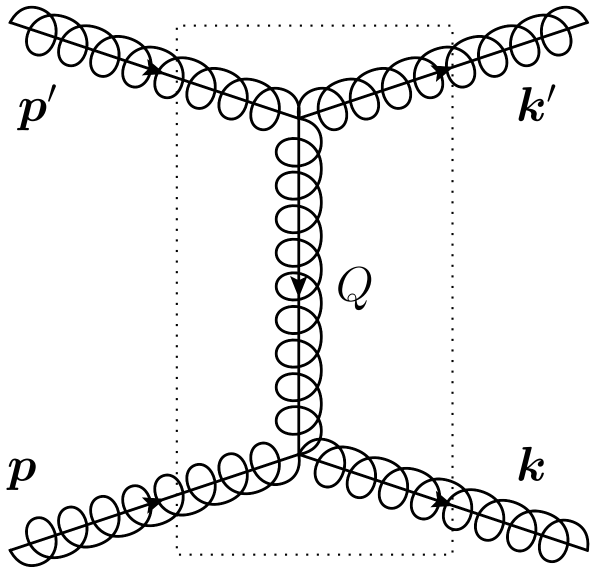
- 1↔2 processes: strictly collinear kinematics, unaffected by reorganization



- Reorganization of the LO collision operator

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left[C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{cross}} + C^{\mathrm{coll}}\right]$$

- Final ingredient: 2↔2 large angle scatterings, IR-regulated to avoid the soft region



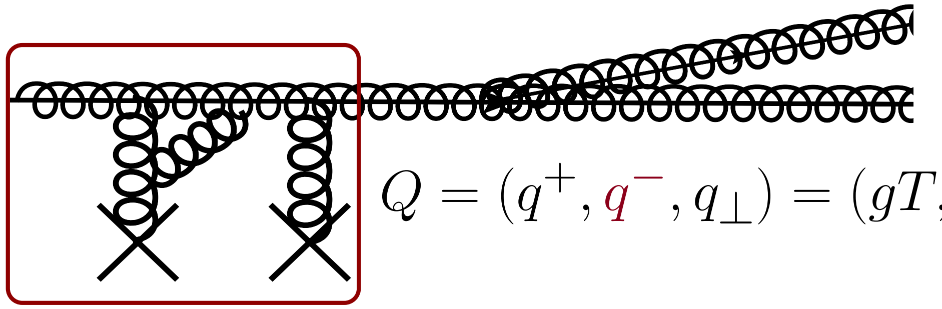
Going to NLO

- The diffusion, cross and collinear terms receive $O(g)$ corrections
- There is a new semi-collinear region

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO $C(q_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

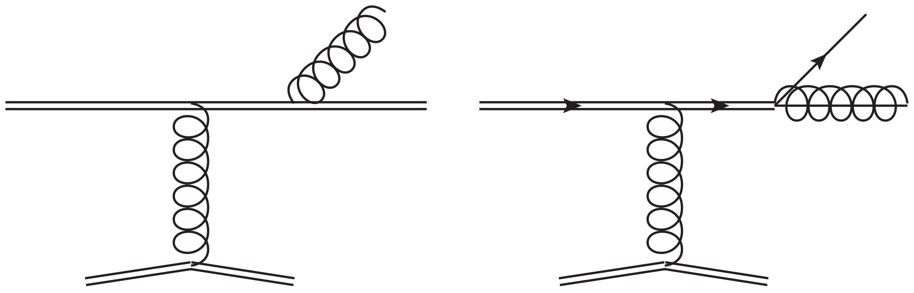
$$\mathcal{C}_{\rm LO}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$



$$Q = (q^+, q^-, q_\perp) = (gT,\, g^2T,\, gT)$$

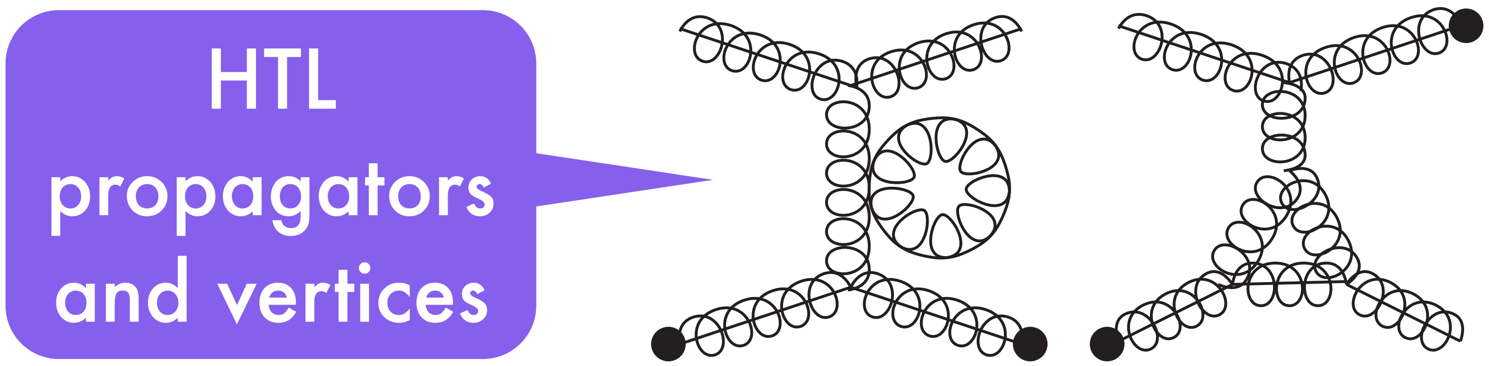
$\mathcal{C}_{\rm NLO}(q_\perp)$ complicated but analytical (Euclidean tech)
Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013)

- Regions of overlap with the diffusion and semi-collinear regions need to be subtracted



NLO diffusion and cross

- At NLO one has these types of diagrams

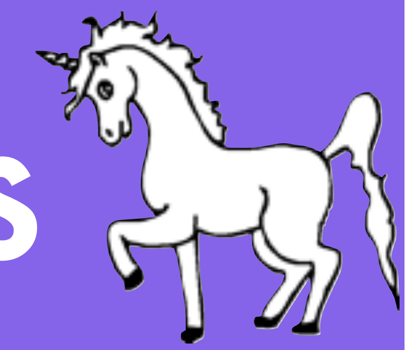


- For diffusion (left): application of light-cone techniques still possible, huge simplification and closed-form results
  Transverse (NLO $\hat{q}$) is finite Caron-Huot (2008)
  Longitudinal (NLO $\hat{q}_L$) is UV log-divergent JG Moore Teaney (2015)

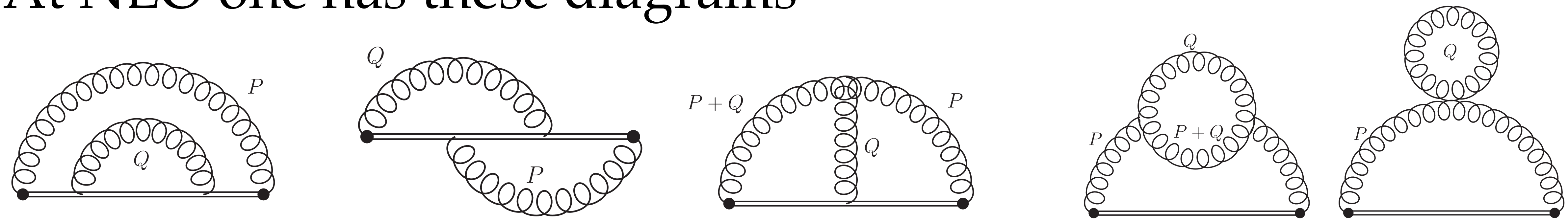
$$\hat{q}_{\rm NLO} = \hat{q}_{\rm LO} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

$$\hat{q}_L(\mu_\perp)_{\rm NLO} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \frac{q_\perp^2\, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}\right]$$

Diffusion corrections



- At NLO one has these diagrams



- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\rm NLO} = \hat{q}_{\rm LO} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

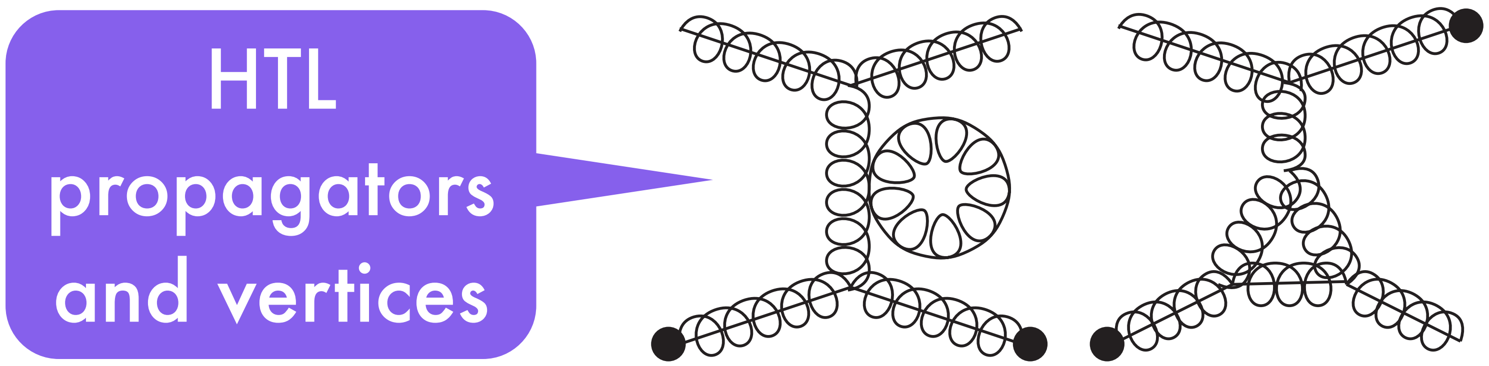
$$\hat{q}_L(\mu_\perp)_{\rm LO} = g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\rm NLO} = g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \boxed{\frac{q_\perp^2\, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}}\right]$$

after collinear subtraction light-cone sum rule still sees only dispersion relation ($O(g)$ correction). NLO still UV-log sensitive

NLO diffusion and cross

- At NLO one has these types of diagrams

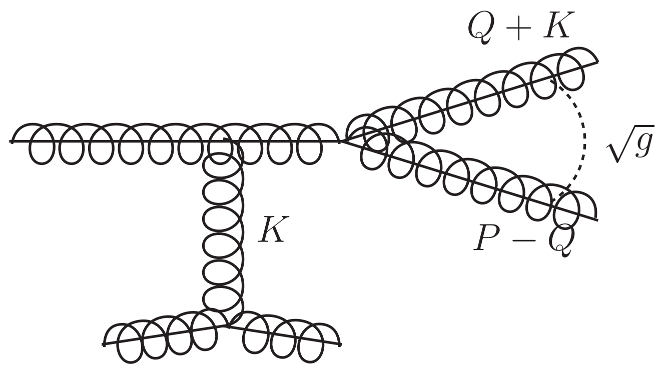


- For cross (right): no diffusion picture = no "easy" light-cone sum rules, only way would be bruteforce HTL. **Missing**, but silver lining: they're finite, so just estimate the number and vary it
  NLO test ansatz: LO cross x $m_D/T(\sim g)$ x arbitrary constant that we vary

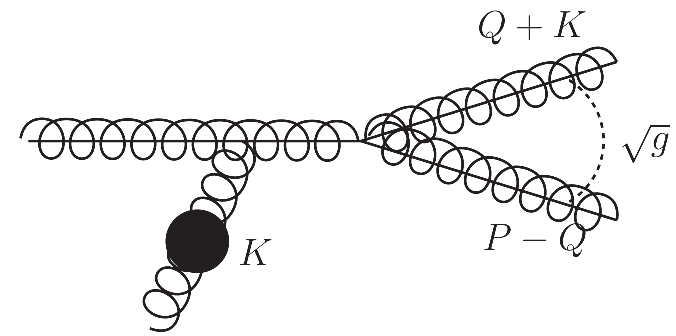
$$C_{\rm NLO}^{\rm cross} = C_{\rm LO}^{\rm cross} \times \frac{m_D}{T} \times c_{\rm cross}$$

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike



K soft plasmon, timelike

- Evaluation: introduce *"modified $\hat{q}$"* tracking the changes in the small light-cone component p^- of the gluons. Can be evaluated in EQCD

"standard" $$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$

"modified" $$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=\delta E}$$

- Rate ∝ *"modified $\hat{q}$"* x DGLAP splitting. IR log divergence makes collision operator finite at NLO

Semi-collinear processes

- Important technical detail: subtractions (no, I am not talking about first grade algebra)
- Pure *O(g)* semicollinear rate actually involves subtraction of collinear and hard limits ,i.e. $\hat{q}(\delta E) - \hat{q}(0) - \hat{q}(\delta E, m_D \to 0)$
- This makes it mostly negative: when extrapolating to larger *g* we risk a negative collision operator
- We devised a new implementation that, while equivalent at *O(g)*, is better behaved when extrapolating due to resummations
- In a nutshell, make $C(q_\perp)$ δE-dependent in the first-order of the LPM ladder resummation. Smoothens *1/k* Bethe-Heitler IR. Implications for thermalization?

Results

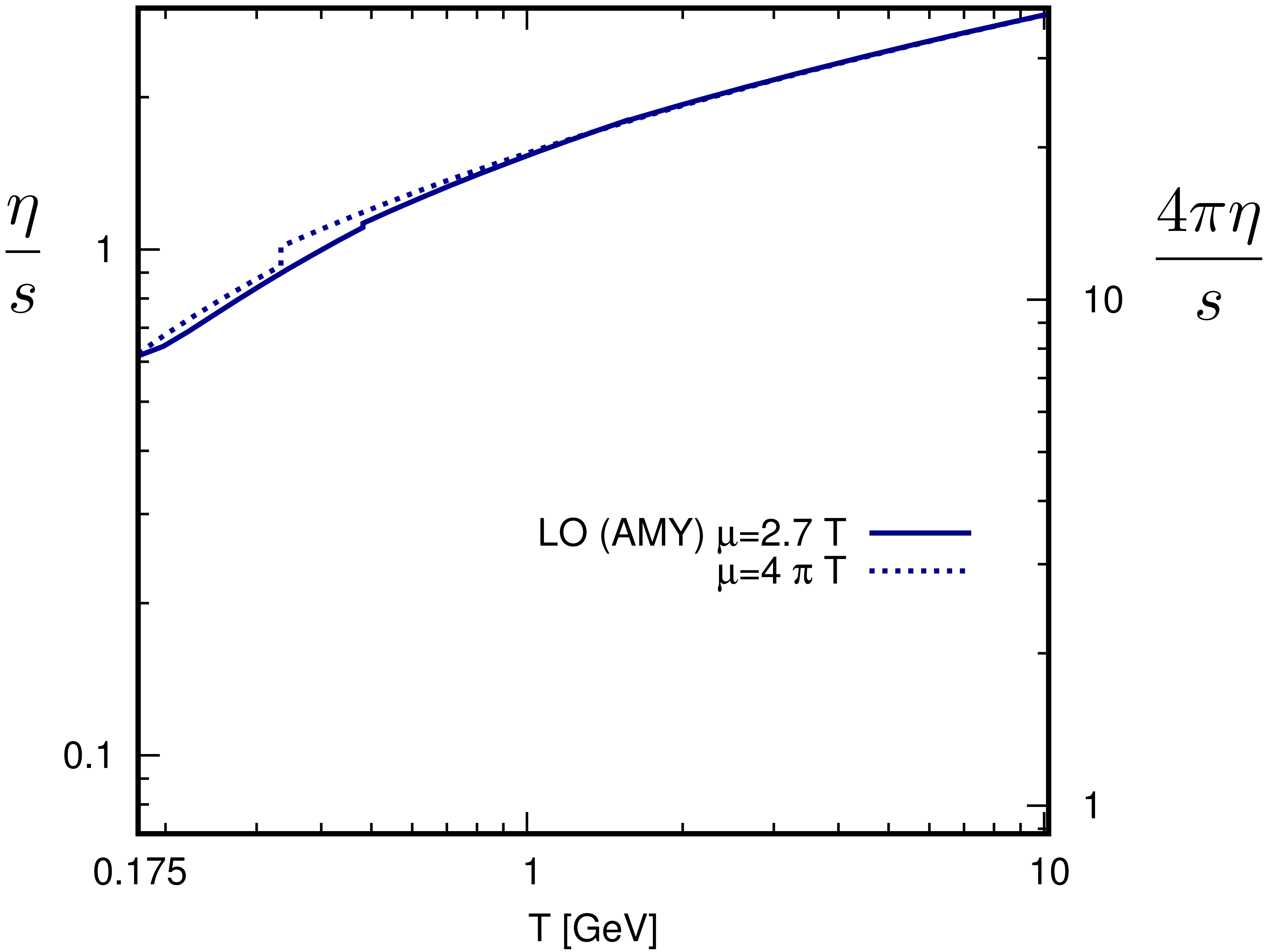
Results

- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion
- Kinetic theory with massless quarks still conformal to NLO
- Relate parameter $m_D/T{\sim}g$ to temperature through two-loop $g(T)$ as in Laine Schröder **JHEP0503** (2005)

Degree of arbitrariness in the choice of quark mass thresholds, test several values of μ/T

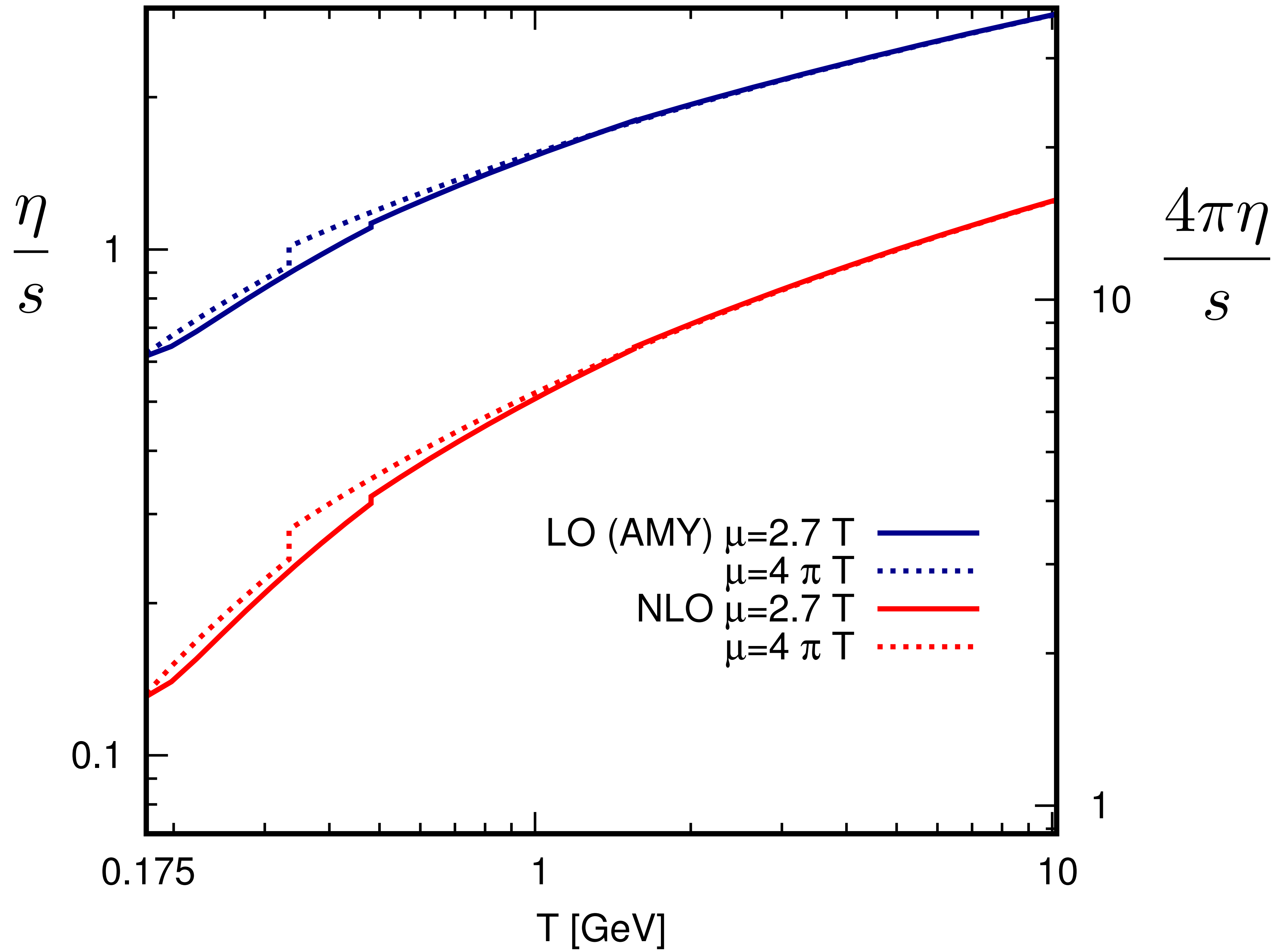
- All plots are **preliminary**

η/s(*T*) of QCD



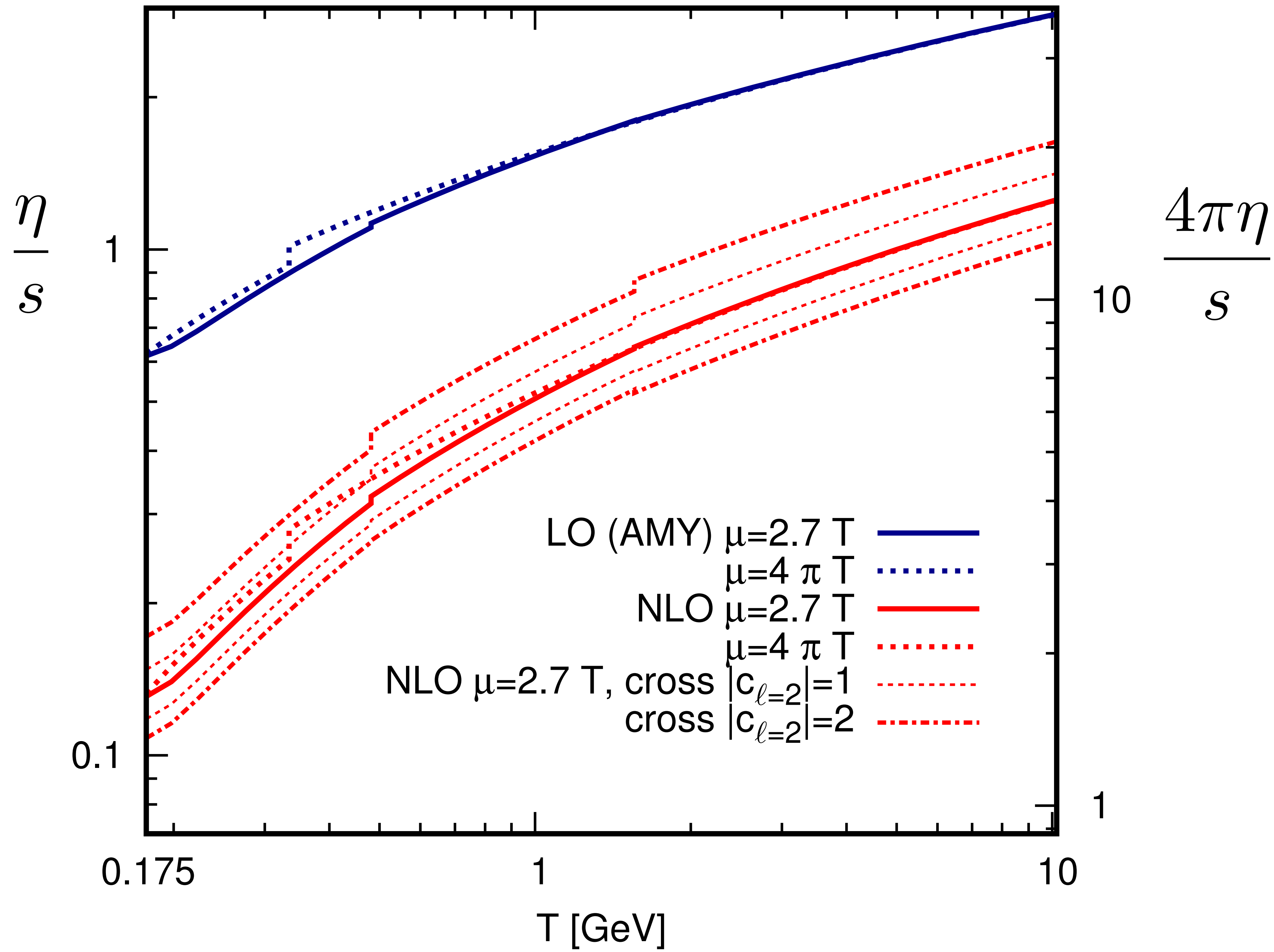
- LO results from AMY (2003)

$\eta/s(T)$ of QCD



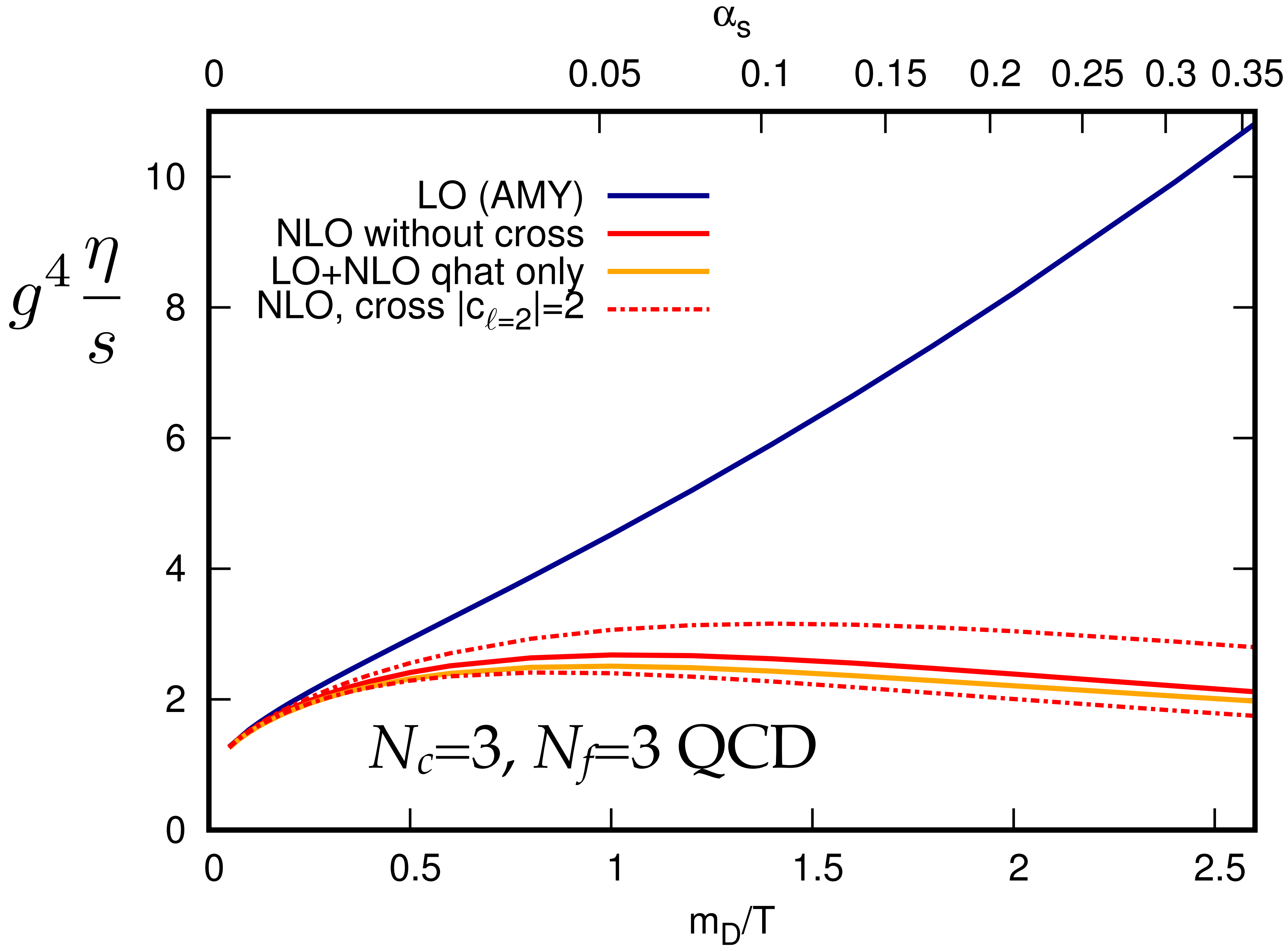
- All known NLO terms, no cross ansatz yet

η/s(*T*) of QCD



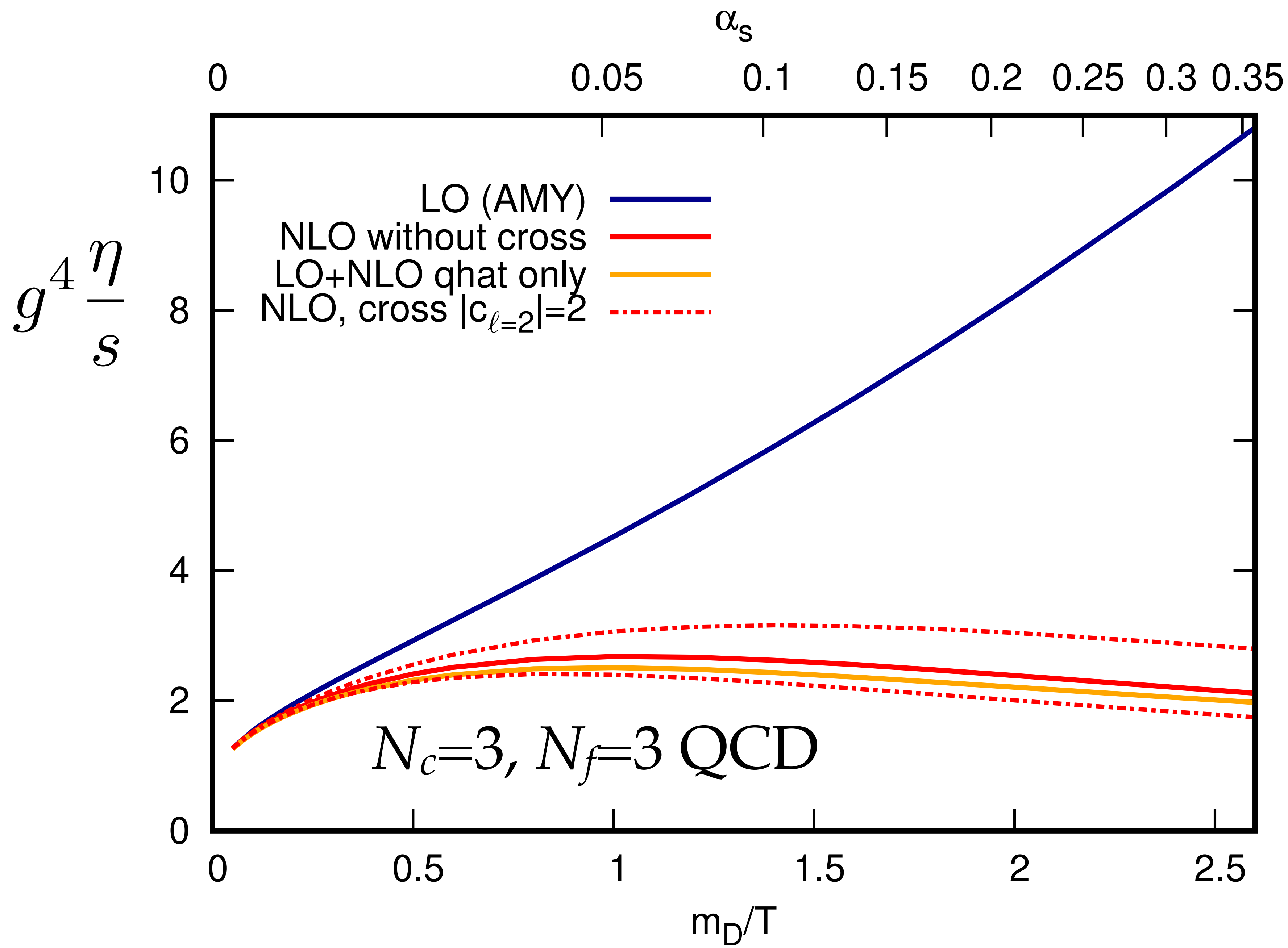
- Cross ansatz introduces $O(\pm 30\%)$ uncertainty

η/s convergence



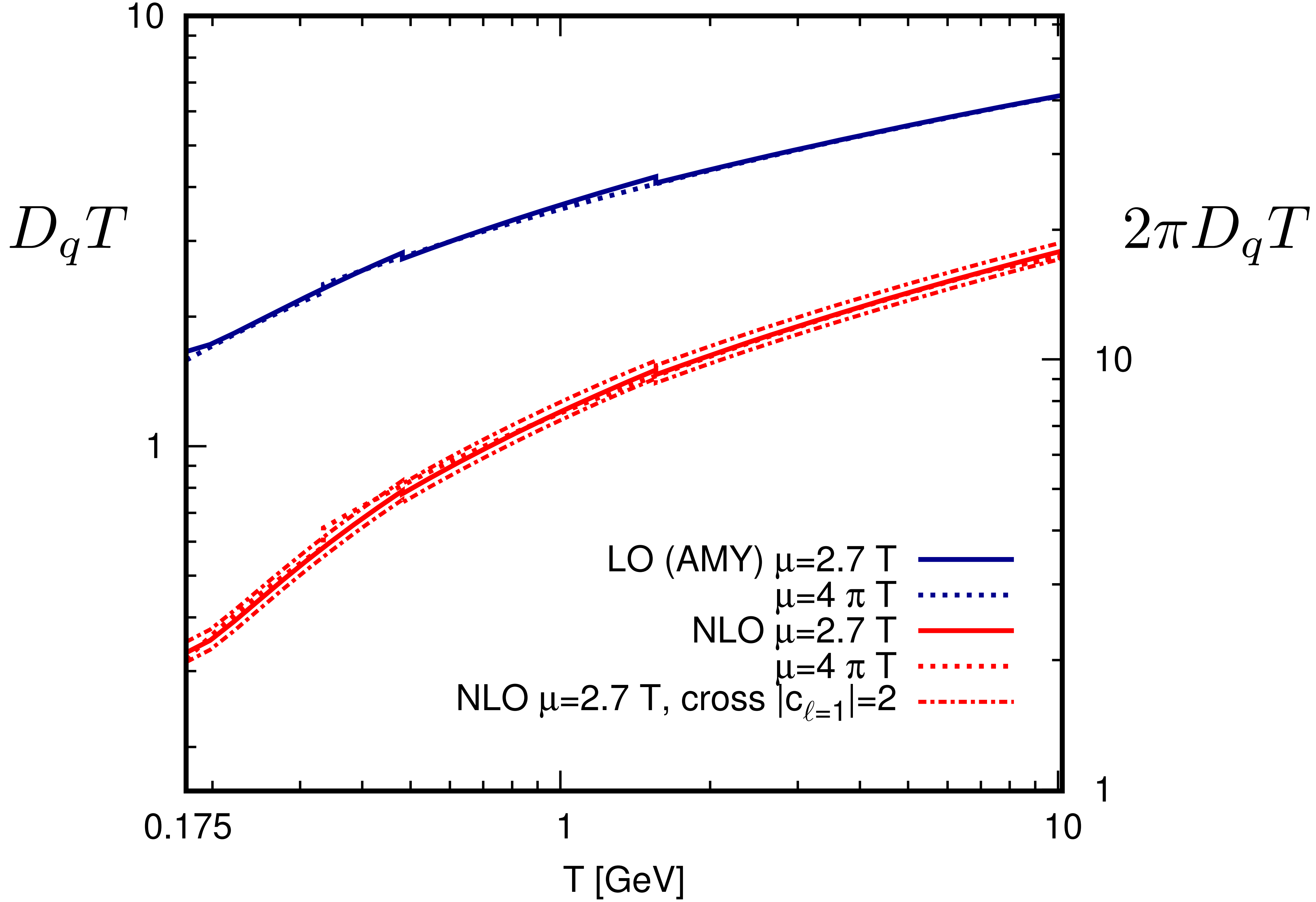
- Convergence realized at m_D~$0.5T$

η/s convergence



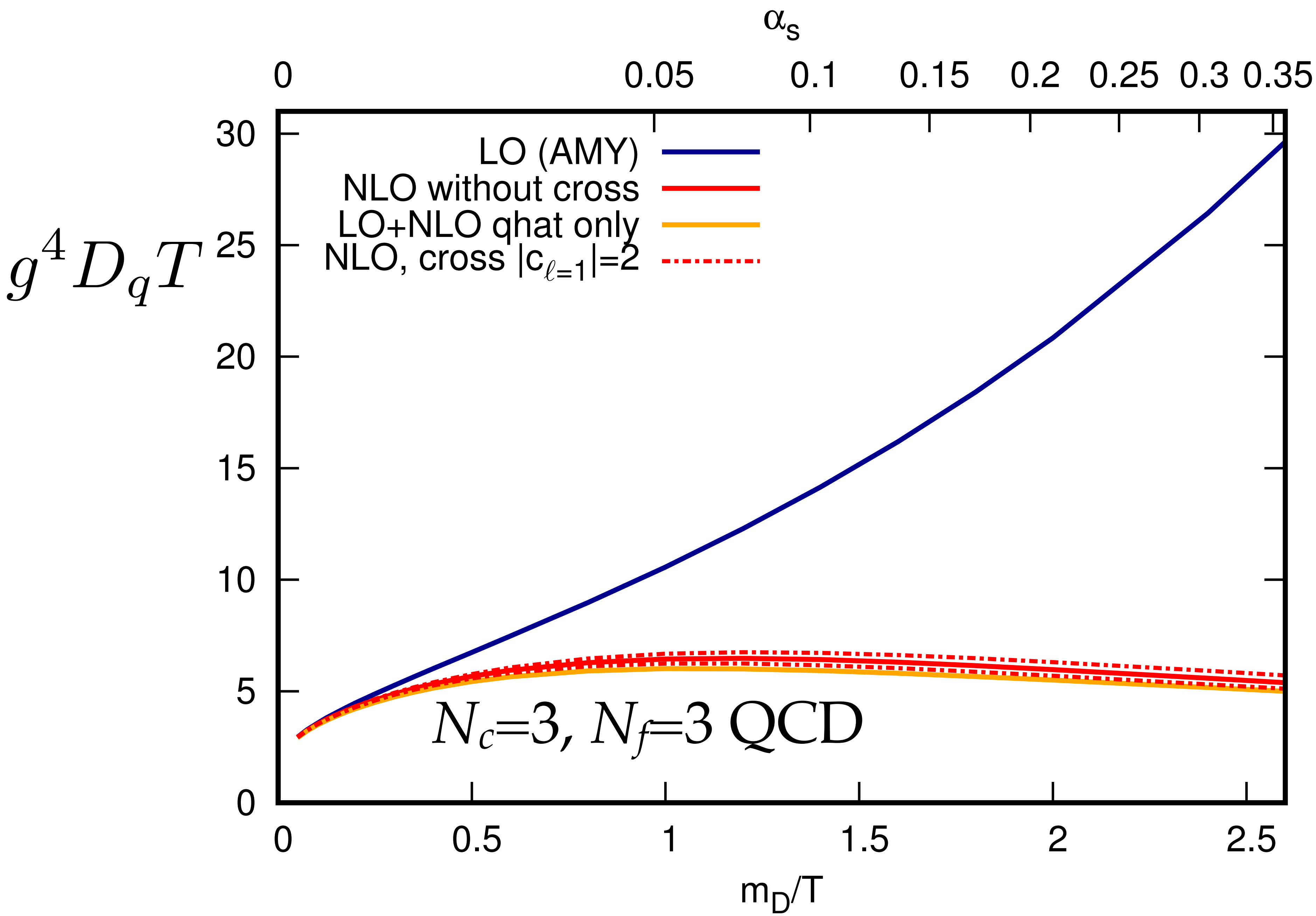
- The ~entirety of the downward shift comes from NLO O(g) corrections to $\hat{q}$

$D_q T(T)$ of QCD



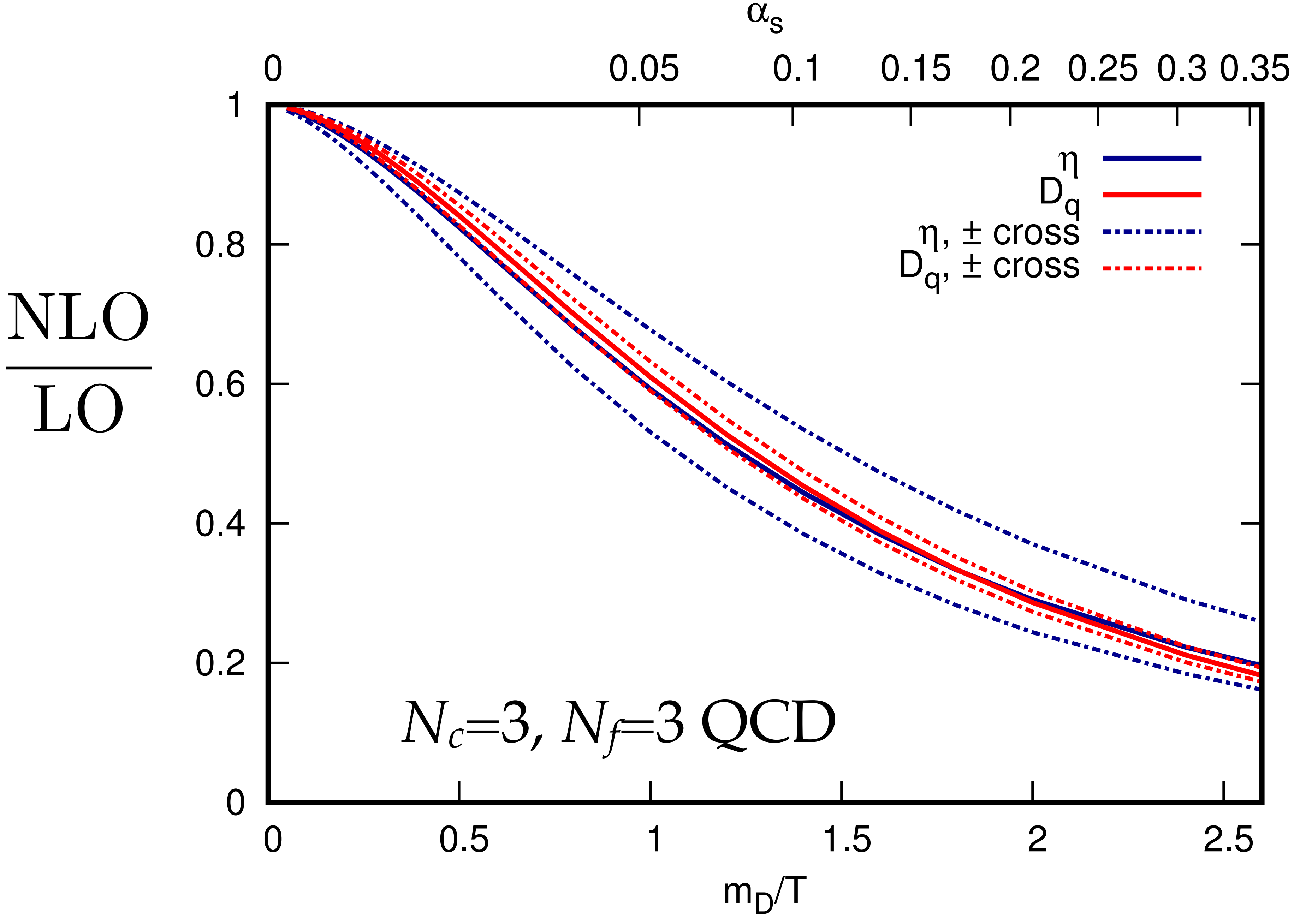
- Cross ansatz uncertainty much smaller (soft quarks here)

D_qT convergence



- Convergence realized again at $m_D \sim 0.5T$

Ratios



- NLO $\hat{q}$ domination makes ratios similar

Conclusions

All those moments will (hopefully not) be lost in time

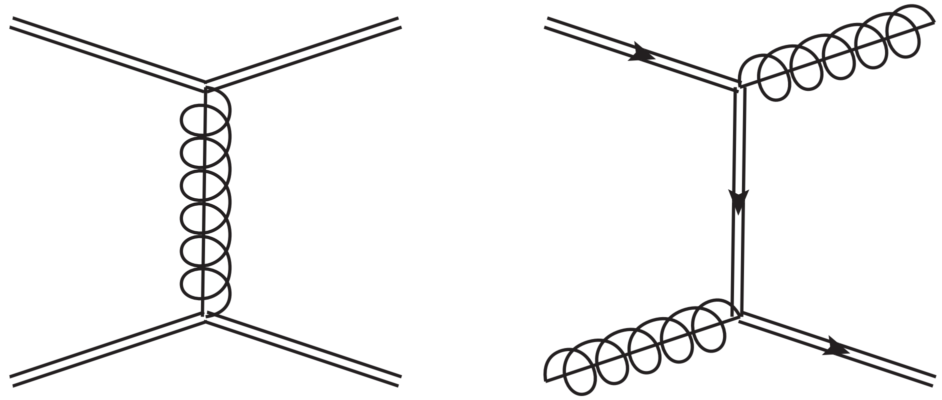
- We have computed all contributions to the NLO linearized collision operator but one (for each ℓ)
- NLO corrections are #large, η and D down by a factor of ~5 in the phenomenological region
- Convergence below m_D~$0.5T$

Second-order τ_Π will be available in the papers

- Corrections dominated by NLO $\hat{q}$. Could it be that observables directly sensitive to transverse momentum broadening show bad convergence and those who are not show good convergence? Why? #statisticswithsmallnumbers

Backup

Elastic processes



Double line: hard (one component O(T) or larger)
Id. specified with curl or arrow when needed

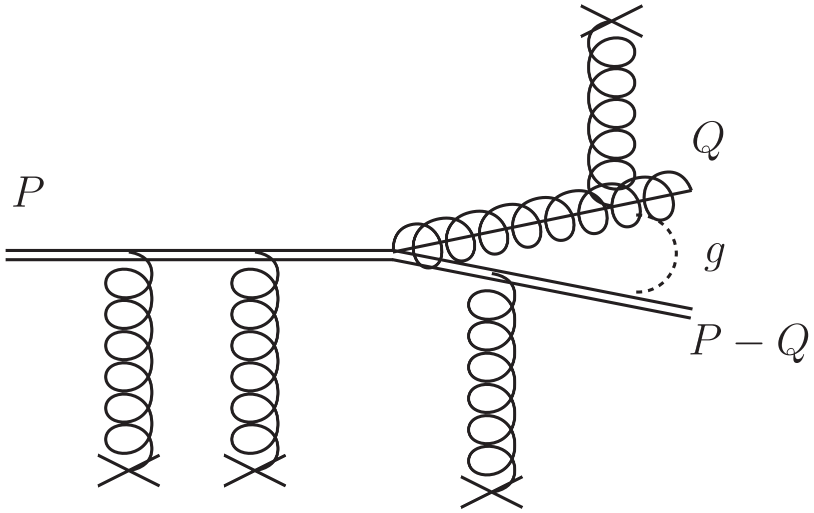
- Boltzmann picture, loss - gain terms

$$C_a^{2\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{1}{4|\boldsymbol{p}|\nu_a}\sum_{bcd}\int_{\boldsymbol{k}\boldsymbol{p}'\boldsymbol{k}'}\left|\mathcal{M}^{ab}_{cd}\right|^2\,(2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\left\{P^a(\boldsymbol{p})\,n^b(k)\,[1\pm n^c(p')]\,[1\pm n^d(k')]-\text{gain}\right\}$$

- Integration with bare matrix elements gives log divergences for soft intermediate states, cured by HTL resummation $\Rightarrow$ nasty n-dimensional numerics?

Radiative processes

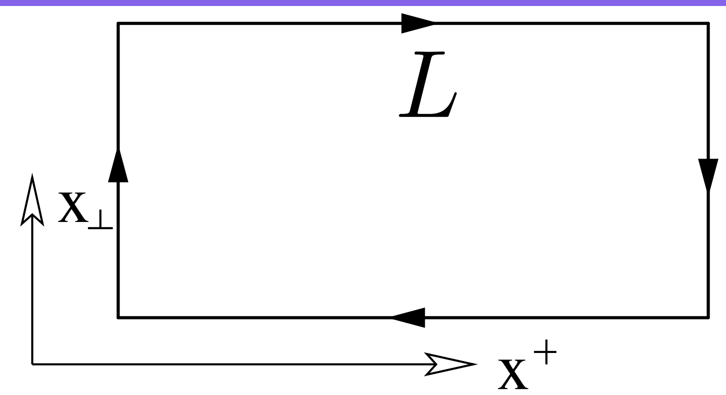
- Effective 1↔2: 1+n↔2+n with LPM suppression, collinear kinematics



$$C_a^{1\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{(2\pi)^3}{|\boldsymbol{p}|^2\nu_a}\left\{ \sum_{bc}\int_0^{p/2} dq\ \gamma^a_{bc}(\boldsymbol{p};(p-q)\hat{\boldsymbol{p}},q\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,[1\pm n^b(p-q)]\,[1\pm n^c(q)] - \mathrm{gain}\Big\}\right.$$
$$\left. + \sum_{bc}\int_0^{\infty} dq\ \gamma^c_{ab}((p+q)\hat{\boldsymbol{p}};\boldsymbol{p},q\,\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,n^b(q)[1\pm n^c(p+q)] - \mathrm{gain}\Big\}\right\}$$

- Rates (gain and loss terms) individually quadratically IR divergent for soft gluon emission/absorption, but gain-loss is finite

Transverse momentum diffusion



$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine Rothkopf JHEP1307 (2013) Panero Rummukainen Schäfer **1307.5850**

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

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- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

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$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0,\mathbf{p})$$

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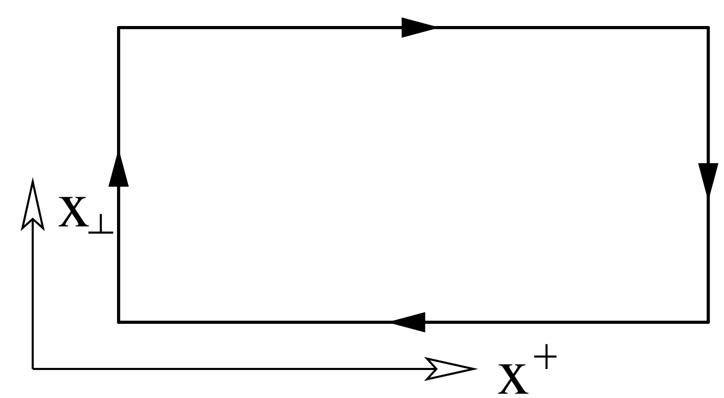
Euclideanization of light-cone soft physics



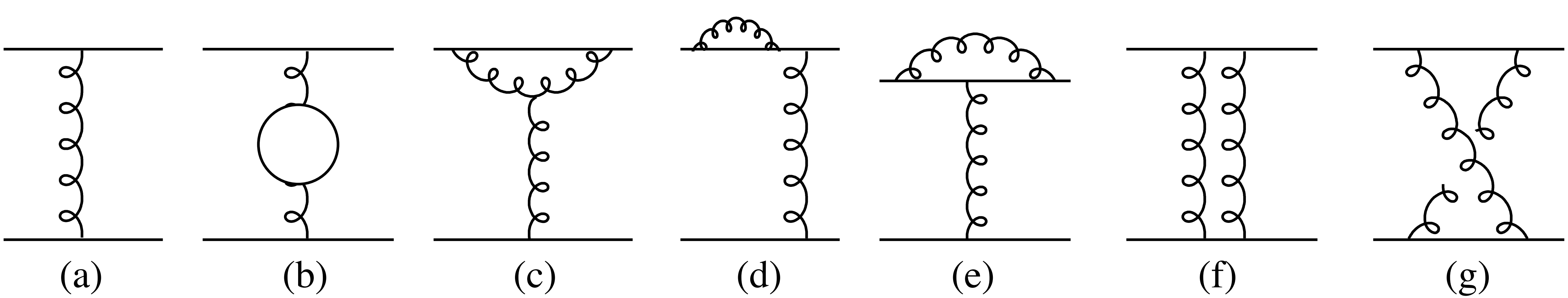
$\propto e^{\mathcal{C}(x_\perp)L}$

- At leading order

$$C(x_\perp) \propto T \int \frac{d^2q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) G_E^{++}(\omega_n = 0, q_z = 0, q_\perp) = T \int \frac{d^2q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) \left(\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)

- At NLO: Caron-Huot **PRD79** (2009)



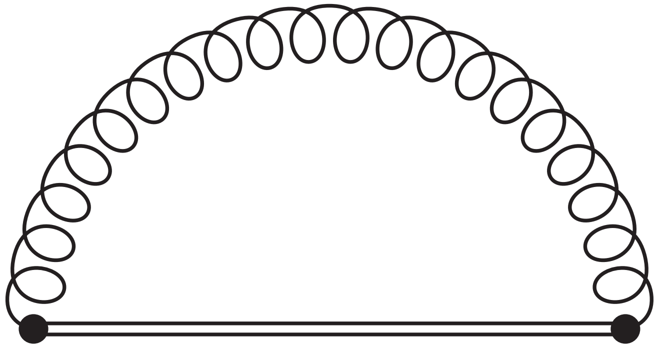
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

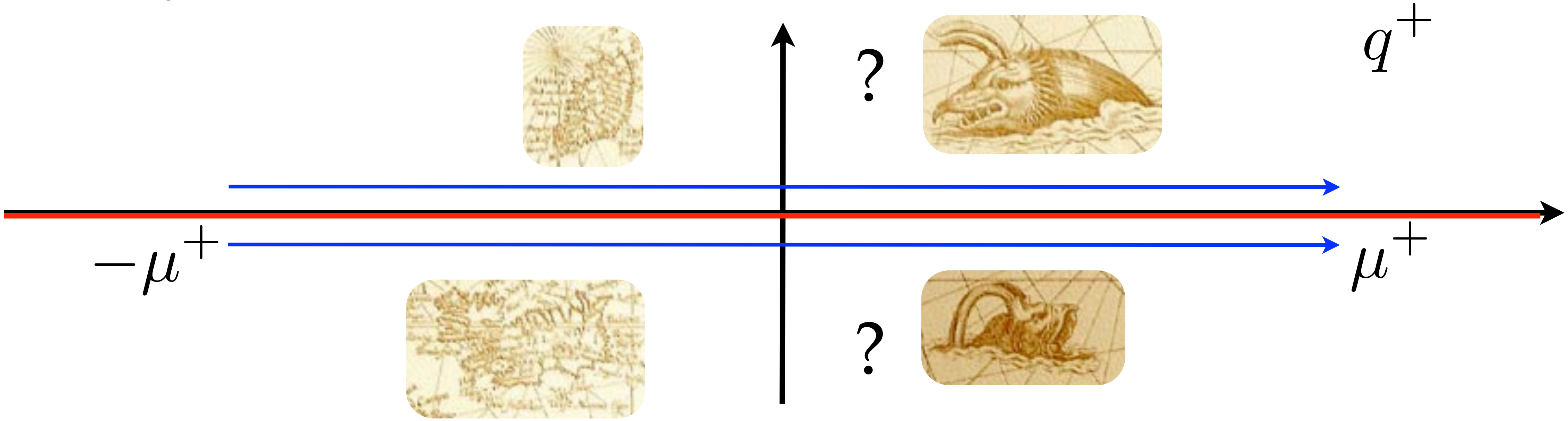


$$\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$$

$$= \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+,q_\perp,0) - G^A)$$

Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



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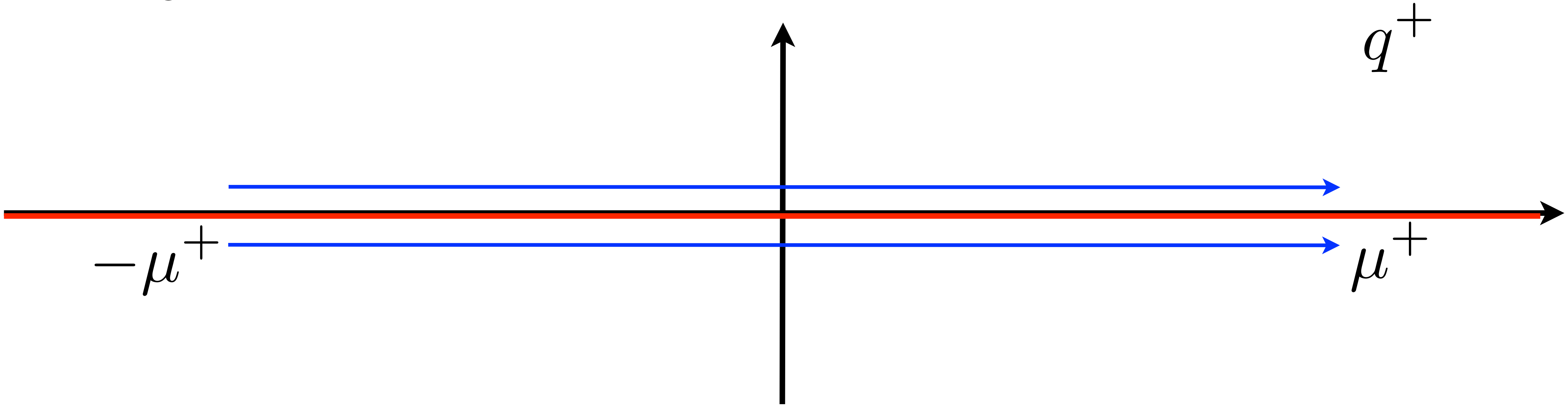

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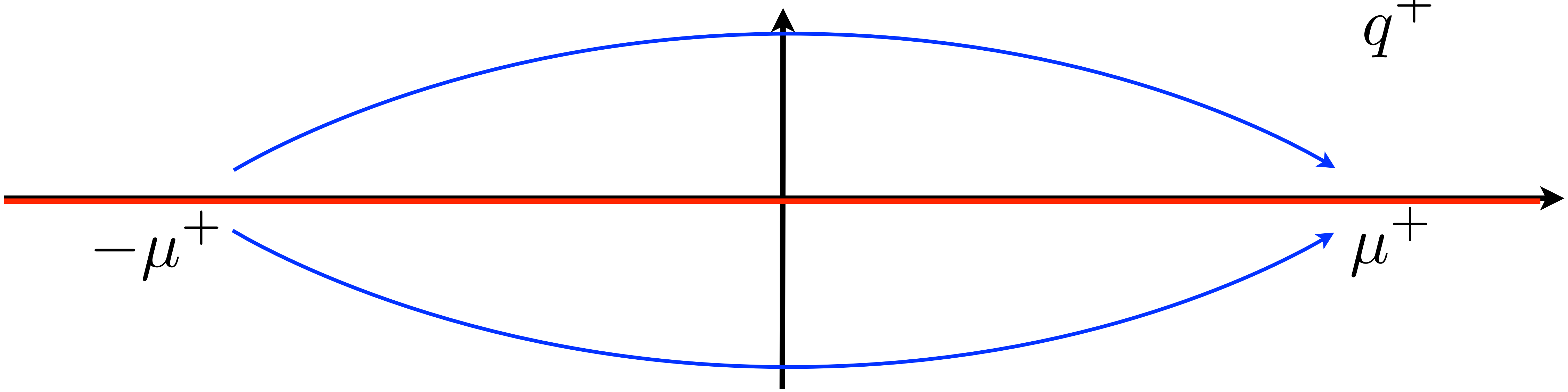

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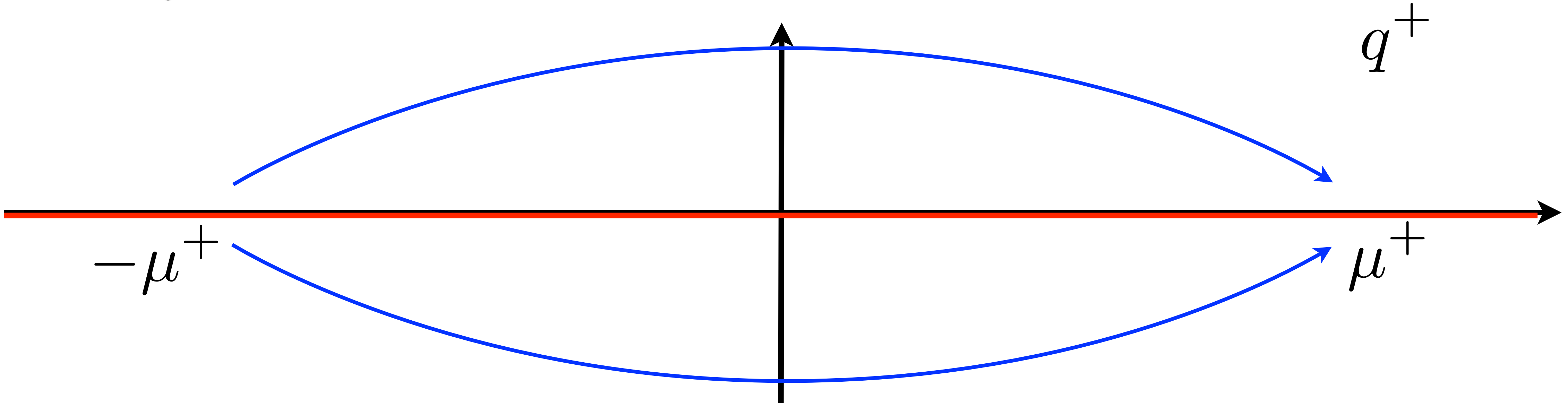

- Use analyticity to deform the contour away from the real axis and keep $1/q^+$ behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$